PAT Maths

Assessment framework

Australian Council *for* Educational Research

ACER

Contents

September 2024

Figures and tables

Introduction

The ACER Progressive Achievement Tests in Mathematics, commonly known as PAT Maths, are a set of assessments that allow teachers to accurately and efficiently measure students' abilities in Mathematics, to diagnose gaps, strengths and weaknesses in student learning, and monitor student progress over time. The assessments have been developed especially, but not exclusively, for use in Australian schools and results can be compared to representative Australian norms at each year level. The PAT Maths construct is appropriate for broad international use and has been adapted to form the basis of a number of assessments developed by ACER for international contexts.

PAT Maths assesses a range of content areas and proficiencies to ensure that the breadth of students' mathematical abilities is captured. The assessments are designed to be engaging and to encourage students to interact with the content to the best of their ability.

A summary of each of ACER's PAT Maths assessments is provided here. More information about the delivery and reporting of the online assessments can be found within ACER's online School Support Centre.

PAT Maths Adaptive

PAT Maths Adaptive (2021) is the most recently developed assessment to use the PAT Maths construct. The assessment comprises a number of testlets (small blocks of test items), of which there are eight different entry levels. In total, students complete three testlets in a sitting.

Entry levels are automatically assigned to students depending on their previous PAT Maths scale score or their current year level. After completing each testlet, students are automatically allocated another group of items of targeted difficulty based on their cumulative performance to that point.

The design of PAT Maths Adaptive allows for more accurate targeting of students' ability levels and removes the need for teachers to select the most appropriate test levels for their students.

PAT Maths 4th Edition

PAT Maths 4th Edition, (2015) comprises test forms ranging from Test 1 to Test 10 and can be administered according to student ability, based on previous scale scores and the educator's professional judgement.

The fixed format, linear construction of these tests allows teachers to compare the performance of a group of students on a shared set of test items.

For more information, please refer to the *Teacher Manual, Progressive Achievement Tests in Mathematics Fourth Edition* (Stephanou & Lindsey, 2013).

PAT Maths Plus

PAT Maths Plus (2010) is a retired online assessment instrument comprising linear test forms ranging from Test 1 to Test 10. The items measured ability on the PAT scale.

PAT Early Years Maths

PAT Early Years Maths (2016–17) comprises four test forms - Start Foundation, Mid-Foundation, Mid-Year 1 and End Year 1, specifically designed for the first two years of formal schooling. These names indicate a suggested time to administer the tests to students and to communicate the progressive difficulty of each test. They are designed for tablet delivery, supported by audio where appropriate and can be completed by students independently at their own pace.

The fixed format, linear construction of these tests allows teachers to compare the performance of a group of students on a shared set of test items.

Rationale for PAT Maths

Numeracy and the application of mathematical skills are considered essential in everyday life. 'Mathematics' is a domain of learning within the Australian Curriculum, and 'Numeracy' is also included in the curriculum as a general capability. There is an expectation that students are able to apply their mathematical understanding across domains. It is therefore essential that students attain a standard of numeracy that enables them to contribute and participate fully at school and beyond, in their adult lives. Monitoring and evaluating mathematical ability is a necessary part of achieving this goal, and PAT Maths provides valid and reliable assessment data to help teachers target where students are in their learning.

Progressive Achievement approach

The Progressive Achievement approach provides a framework for integrating student assessment, resources that support teaching practice, and professional learning. PAT assessments allow teachers to collect evidence of student learning; to identify where students are in their learning at a given point in time; to monitor growth over time; and to reflect on student attainment. They provide reliable measures that enable a variety of interpretations about attainment and progress, such as:

- what students attaining specific levels of progression are likely to know, understand and be able to do;
- how much students have improved over time and what skills, knowledge and abilities they have been able to develop; and
- how a student's level of attainment compares with other students'.

The value of an integrated approach to assessment and student learning has become widely acknowledged. There is now a wide variety of formative, diagnostic assessment tools used in Australian classrooms. Summative assessments, such as NAPLAN, are also often used to inform teaching and learning. As Dylan Wiliam (2011, p. 43) makes clear, 'any assessment is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction'. In his Report, David Gonski (2018, Finding 7) refers to the compelling evidence that 'tailored teaching based on ongoing formative assessment and feedback is the key to enabling students to progress to higher levels of achievement.'

ACER's PAT tests provide indicators of student achievement via scale scores and the accompanying achievement band descriptions. Upon completing their assessments, students are allocated a scale score that represents their ability in mathematics. The PAT scale is divided into achievement bands from which the skills and understanding represented at each level are described. The achievement bands provide valuable evidence-based information about the concepts and skills students have achieved, are consolidating and are working towards. As the Gonski report recommends, reporting on assessment should have an emphasis on achievement and growth and that growth should be measured against learning progressions (Gonski, 2018, Recommendation 4). Masters (2013) also expresses the idea that learning should be assessed by measuring growth over time and against empirically derived learning progressions.

The PAT reports provide targeted formative feedback, allowing the student data to be sorted and analysed in a variety of ways. Using the PAT data and the achievement band descriptions, teachers can structure learning specifically to students' needs rather than where they are expected to be.

Progressive Achievement in Maths

The connection between curriculum, practice and assessment must be strong for any one element to be effective (OECD, 2013) and so it is important that the assessment is based on what is taught and that the results of any assessment inform the teaching that follows. Assessment should be more than just the instrument; it includes 'the process of drawing inferences from the data collected and acting upon those judgements in effective ways' (Callingham, 2010, p 39). Clarke (1996) suggests that teachers should ask themselves, 'how will this assessment promote and inform subsequent action by me, by other teachers, by my students and by parents or other members of the community?'

The PAT Maths assessment is designed to target the key skills and concepts that underpin growth, and to assist teachers in understanding the progression of mathematical understanding through the PAT reports and supporting resources. Items cover content strands (Number and algebra, Measurement and geometry, and Statistics and probability) and proficiencies (Understanding, Fluency, Problem solving, and Reasoning). The assessment includes items that range in difficulty and feedback is provided immediately through a series of reports that allow teachers to analyse group and individual performance by content strand, proficiency and item difficulty. PAT teaching resources are organised by content strands at the relevant achievement band level and can be used for differentiated classroom practice and to monitor more specific areas of growth.

While growth indicators are available for many formative assessment tools, the PAT Maths achievement band descriptions are evidence-based, developed from valid and reliable assessment data that has identified a 'typical' trajectory of development. This can provide teachers with confidence in the data they are using to target areas of learning, and to identify how students progress over time.

PAT Maths and curricula

The Australian national curriculum, and all state curricula, describe expected levels of performance in mathematics and numeracy based on year or stage level. Numeracy is a general capability in the Australian Curriculum and is embedded throughout the curriculum across domains.

PAT Maths items are not explicitly developed according to the Australian Curriculum, as the PAT construct is based on a Progressive Achievement approach, rather than year-based expectations. PAT Maths results do not directly align with curriculum-based year or stage level outcomes, but items are mapped to the Australian Curriculum and some state curricula, with content codes and descriptions provided in the online reports and the PAT Teaching Resources Centre.

PAT Maths assesses mathematical skills and applications in more depth than curricula, which generally describe the skills in broader terms. For this reason, there is often a single content description aligned to many test items at similar levels of difficulty. A single assessment is likely to be aligned to curriculum descriptions across a range of year levels, because each test assesses a range of ability.

Construct

Definition

A construct is a description of an ability that can be measured on a single dimension (with a single numeric variable). It often refers to 'what students know and can do'. A mathematical model is used to transform observations (eg student responses to test items) into measurements. A careful definition of ability/proficiency helps ensure that the assessment and reporting are consistent and legitimate.

PAT Maths aims to measure mathematical ability. It conceives of ability in mathematics as being more than just remembering mathematical facts and procedures. Students must be able to use their mathematical knowledge to solve problems, interpret data, and support or refute claims. To be able to do so, students must have a solid foundation in the basic skills and so must be able to quickly and accurately perform routine calculations. With these considerations in mind, the definition of mathematics that is used is as follows:

Mathematical ability *is the facility with which a person can: retrieve mathematics facts, procedures and concepts; apply them in both abstract and practical contexts to solve problems; interpret mathematical data; and give a mathematical argument to support or counter a claim.*

Structure

The PAT Maths construct is the organising principle of the assessments; it is used to guide test development and structure the PAT reports. This structure is also part of the Progressive Achievement approach because the knowledge, skills and understanding represented in the tests are designed to support educators in identifying student needs.

Three overarching elements guide assessment development:

- Strands
- Proficiencies
- Contexts

Strands

It is helpful to divide the domain of mathematics into topic areas or content strands. When developing test items, each can be targeted on one strand.

In the Australian Curriculum, Version 9.0 there are six strands: Number, Algebra, Measurement, Space, Statistics and Probability. In PAT Maths and the ACER Learning Progressions, three strands are used: Number and algebra, Measurement and space, Statistics and probability. These are also the basis of the sub-strands in PAT Maths. There are eight sub-strands in PAT Maths:

- Whole number operations
- Fractions and decimals
- Money and financial maths
- Patterns and algebra
- Measurement
- Space
- Statistics
- Probability

Progression in the strands and sub-strands is reflected by the achievement band descriptions, as discussed in the section Reporting.

Whole number operations

This sub-strand comprises the four basic arithmetic operations of addition, subtraction, multiplication and division. At the lower levels, especially, it also includes the underlying ideas of whole numbers such as number sense, counting principles, place value, ordering and comparing. These are the essential understandings that form the basis of much of the mathematics that people are called to do in their everyday life. Students need to be able to recognise situations in which operations are relevant and work with these operations flexibly and efficiently.

Fractions and decimals

This sub-strand involves working with quantities expressed as fractions, decimals or percentages, and the relationships between these. At higher levels this can include ratios and proportional reasoning, index notation and surds. At the lower levels it includes dividing groups and single objects into equal parts.

Money and financial mathematics

This sub-strand refers to students' proficiency in handling Australian currency (mainly coins, but also some notes). It also refers to students' familiarity and proficiency with financial transactions relevant to everyday life. At the lower levels, this is mainly within the context of role play in making purchases, identifying coins and notes and solving calculations using whole dollars. At the upper levels, contexts extend to profit and loss, percentage discounts, and saving and borrowing, including ideas of simple and compound interest.

Patterns and algebra

At lower levels, this sub-strand involves identifying, copying and continuing patterns using objects, numbers, letters and symbols. This is followed by finding missing numbers in simple patterns and number sequences and number sentences. As students progress up the scale, they extend visual and numerical sequences to find future terms; describe rules for given sequences; use pronumerals in place of unknown values or variables; work with linear and simple quadratic expressions; solve linear equations; expand and factorise linear and simple quadratic expressions; and describe relationships using words, symbols, tables and graphs.

Measurement

Measurement involves the concepts of length, area, volume, capacity, mass, time and temperature.

Important skills in measurement include the ability to use clocks, timetables and calendars to solve problems; the ability to read and interpret a variety of different types of measurement scales and convert between metric units; and the exploration of relationships between length, area and volume for a variety of shapes and objects, including the application of these relationships to solve problems in context.

At lower levels, measurement involves descriptions of attributes, comparing objects and the use of informal units. At the higher levels, proportional reasoning is an essential cognitive skill in order to effectively solve problems involving circle properties and trigonometric ratios.

Space

Space comprises several key ideas:

- the concept of shape, referring to the recognition, visualisation and description of key features of common two-dimensional shapes and three-dimensional objects;
- navigation through a space, including the ability to use language to describe direction and position; and the ability to read, interpret and construct maps at varying levels of complexity and detail;
- symmetry, including an understanding of the main symmetrical relationships (reflection, rotation, translation) in a variety of contexts (the ideas of similarity and the enlargement transformation are also introduced at higher levels);
- the relationship between line and angle, in particular, the angle relationships associated with triangles, quadrilaterals, parallel lines and transversals.

Statistics

In Statistics, students work with collecting, understanding and comparing data. Important skills include interpreting and using both categorical and numerical data. At lower levels, this involves students collecting, sorting and comparing objects, recording data using tally marks and creating one-to-one displays. As complexity and difficulty further increase, students interpret and describe general trends in a data set and can interpret and calculate measures of range and central tendency.

Probability

At lower levels, Probability involves being able to determine the likelihood of familiar, everyday chance events using very simple descriptions and informal terms such as 'even chance', 'good chance', and words such as 'impossible', 'unlikely', 'likely' and 'certain'. As students progress in their understanding, they begin to assign numerical values (fractions) to describe chance for simple experiments such as tossing a coin, picking an object from a bag or rolling a die. With further progress, equivalent forms for assigning probabilities, such as decimals and percentages, are also recognised and the complexity of events and experiments explored is gradually increased from simple to compound events, or one-step to multi-step processes. At higher levels of complexity, Probability involves the use of a wider variety of structured diagrams and tables such as Venn diagrams, two-way tables and tree diagrams to list the sample space of a given experiment and explore and analyse the outcomes of probability experiments.

Proficiencies

The ACER Learning Progression considers four areas of in the development of understanding and skill in Mathematics - conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. PAT Maths also addresses the mathematical proficiencies identified in the Australian Curriculum:

- Understanding
- Fluency
- Problem solving
- Reasoning

Each PAT Maths test item is mapped to a proficiency. It is noted that these proficiencies are not discrete and an individual item will likely call on more than one of the four. For classification, we take the main demand required by an item. The four proficiencies are described in the Australian Curriculum as follows:

Understanding

Students build and refine a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically, and when they interpret mathematical information.

Fluency

Students develop, practise and consolidate skills in choosing appropriate procedures; carrying out procedures flexibly, accurately, efficiently and appropriately; and recalling factual knowledge and concepts readily. Students are fluent when they use conceptual understanding to calculate answers efficiently, robust ways of answering questions, and choose appropriate methods and approximations. Students who display fluency can recall definitions and regularly use facts and can manipulate expressions and equations to find solutions.

Problem solving

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

Reasoning

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying, and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices.

Contexts

Test items are each associated with a context type. A context is the situation within which the details of a test item or task are located, or the situation that generated the stimulus material for the task. Contexts help to define the focus of thought or action in which persons responding to problems or challenges must engage.

Contexts may be abstract or practical. Questions related to Problem Solving and Reasoning predominantly have practical contexts whereas Fluency and Understanding are assessed more abstractly.

Abstract

Abstract problems are suited to pinpointing whether a student knows a particular piece of mathematical knowledge or a mathematical procedure. The lack of a real-world context enables students to concentrate purely on the knowledge or procedure required to solve the problem. The amount of reading required to understand context-free questions is minimal, so all students have a chance of displaying what they know and can do.

Practical

Problems with a practical context might require a student to translate a situation described in everyday language into a mathematically-posed problem and, after solving this problem, perhaps to interpret the solution in terms of the original context. Because of this, students often find practical contextual problems more challenging. Care must be taken in writing such items so as not to unintentionally assess knowledge of the context instead of knowledge of mathematics, nor to have a reading load such that it affects the student's ability to solve the problem. This risk is addressed in PAT Early Years with a 'real aloud' function. Using a variety of contexts can help mitigate this problem. Practical contexts can be further subdivided as having an individual focus, an interactive focus, and a wider-world, or external, focus. Early years contexts predominantly include everyday situations both in and out if school.

Individual: The problem or challenge primarily affects the individual and engagement with the task involves an inward focus. For example, challenges might focus on games and puzzles, personal health, personal transport or travel, or personal finance.

Interactive: Practical contexts might also have an interactive focus requiring engagement with other individuals or with elements of the immediately surrounding environment. Problems fitting this description involve day-to-day situations and activities at home or at school, or in the local community, or at work, where the focus of thought and action lies in connections and interactions with immediately surrounding people or objects.

External: Wider-world contexts have an external focus on broader situations that may affect whole communities or the whole country, or that have a wider relevance at a global level. Problems fitting this context type involve broad social issues such as public policy, transport systems, advertising, and broad scientific issues such as weather, climate, ecology or medicine.

Assessment design

Measuring the construct

In developing items and designing the tests, the major criteria considered are as follows:

- distribution of items across strands
- distribution of items across proficiencies
- distribution of item difficulty
- curriculum appropriateness

Distribution by strand and proficiency

It is necessary to assess students on an appropriate distribution of strands so that the assessment encompasses a range of mathematical skill and knowledge. This approach ensures that the formative data gained provides insight into possible strengths, gaps and weaknesses in different areas. The strands and sub-strands are not evenly distributed across all levels because some are more appropriate at the lower levels than the higher and vice versa. Algebra and Probability, for example, require some foundational understanding in Number, and so are not introduced at the earliest levels.

It is also important that the assessments contain items requiring a range of cognitive skills, or proficiencies. This allows educators to explore the ability of students to generalise, synthesise and solve problems as well as demonstrate understanding and fluency. PAT Early Years Maths and PAT Maths 4th Edition were released prior to the inclusion of proficiencies in the Australian Curriculum. Item proficiencies were therefore not considered as part of the development process but have been retrospectively mapped following the release of PAT Maths Adaptive.

Table 1 shows the distribution of items by content strand and proficiency in each of the PAT Early Years Maths tests. Between 50% and 65% of items assess Number and algebra, between 20% and 35% assess Measurement and space and between less than 1% and 20% assess Statistics and probability. Table 2 shows the distribution of items by content strand and proficiency in each of the PAT Maths 4th Edition tests.

Table 1 *Percentages of PAT Early Years Maths items by strand and proficiency for each test*

	Strand %			Proficiency %			
Test level	Number and algebra	Measurement and space	Statistics and probability	Understanding	Fluency	Problem solving	Reasoning
Start Foundation	64	35	<1	27	27	27	19
Mid-Foundation	53	33	14	20	40	10	30
Mid-Year 1	60	23	17	13	37	3	47
End Year 1	57	30	13	10	23	37	30

Table 2 *Percentages of PAT Maths 4th Edition items by strand and proficiency for each test*

	Strand %			Proficiency %			
Test level	Number and algebra	Measurement and space	Statistics and probability	Understanding	Fluency	Problem solving	Reasoning
Test 1	49	34	17	27	23	37	13
Test 2	52	31	17	21	17	48	14
Test 3	52	34	14	31	37	20	11
Test 4	41	31	28	29	37	14	20
Test 5	44	28	28	29	40	20	11
Test 6	43	33	24	30	33	15	23
Test 7	43	33	24	23	25	20	33
Test 8	37	33	30	10	23	33	35
Test 9	40	34	26	13	28	25	35
Test 10	42	32	26	13	35	30	23

Number and algebra items comprise around half of the items in any PAT Maths Adaptive pathway. At the lower levels, between 30% and 40% of the items assess Measurement and space, and between 10% and 20% of the items assess Statistics and probability. At the upper levels, there are at most 35% Measurement and space and 25% Statistics and probability items. These proportions reflect the changing nature of content and curriculum. For instance, Probability is not extensively taught at earlier years.

The proficiencies are not equally distributed within each PAT Maths Adaptive testlet, but there is a balance across pathways that is appropriate to the level. At the lower levels, up to half of the items assess Understanding, 25% Fluency, 25% Problem solving and between 5% and 15% Reasoning. Reasoning is more difficult and so there are fewer reasoning items at the lower levels. For more difficult pathways, these proportions change to reflect the changing nature of the items. At the very upper levels, Reasoning items make up between 20% and 30% of the pathway, whereas Understanding is around 20%, Fluency and Problem solving are both around 30% of each pathway. Table 3 shows the average distribution of item strands and proficiencies across testlets at all stages and levels. Although these figures are representative of each testlet individually, the characteristics of the complete test pathways that students complete will vary according to the level of the three testlets that make up the pathway.

Table 3 *Percentages of PAT Maths Adaptive items by strand and proficiency for each level and stage*

		Strand %			Proficiency %			
Test level		Number and algebra	Measurement and space	Statistics and probability	Understanding	Fluency	Problem solving	Reasoning
Stage 1	Level 1	50	33	17	33	30	20	17
	Level 2	61	26	13	29	29	23	19
	Level 3	53	29	18	29	26	32	13
	Level 4	43	37	20	26	31	20	23
	Level 5	53	26	21	29	24	21	26
	Level 6	45	35	20	23	33	20	25
	Level 7	46	31	23	10	28	26	36
	Level 8	44	31	26	18	28	31	23
Stage 2	Level 1	45	42	13	39	23	23	16
	Level 2	48	29	23	32	29	23	16
	Level 3	46	30	24	22	27	24	27
	Level 4	53	26	21	24	39	16	21
	Level 5	50	31	19	14	31	33	22
	Level 6	50	30	20	20	30	30	20
	Level 7	54	27	20	15	27	22	37
	Level 8	42	33	25	17	22	31	31
	Level 9	57	19	24	16	30	32	22
Stage 3	Level 1	45	34	21	27	34	22	17
	Level 2	48	32	19	35	29	19	16
	Level 3	50	33	17	28	31	19	22
	Level 4	45	33	21	24	36	18	21
	Level 5	42	33	25	19	31	22	28
	Level 6	49	28	23	18	38	26	18
	Level 7	53	25	23	15	35	28	23
	Level 8	51	32	16	16	27	24	32
	Level 9	44	31	26	13	33	18	36

Distribution of item difficulty

It is important to have a spread of item difficulties that matches the abilities of the students. This is especially important in the context of a computer adaptive test to enable efficient convergence of the algorithm.

Table 3 shows the mean difficulty of the items in each of the PAT Early Years Maths tests in scale score units, with their standard deviations. Standard deviation measures the amount of variation in item difficulty for a set of items.

Table 4 *Mean difficulty and standard deviation of each PAT Early Years Maths test*

Test level	No. of items	Mean item difficulty (scale score)	Standard deviation (scale score)
Start Foundation	30	73.2	10.4
Mid-Foundation	30	90.7	12.3
Mid-Year 1	30	96.9	12.5
End Year 1	30	101.5	8.5

Table 4 shows the mean difficulty of the items in each of the PAT Maths 4th Edition tests in scale score units, with their standard deviations.

Table 5 *Mean difficulty and standard deviation of each PAT Maths 4th Edition test*

Test level	No. of items	Mean item difficulty (scale score)	Standard deviation (scale score)
Test 1	30	98.1	6.8
Test 2	29	105.4	7.0
Test 3	35	106.7	10.3
Test 4	35	119.4	8.5
Test 5	35	122.7	7.4
Test 6	40	128.9	7.9
Test 7	40	129.5	8.2
Test 8	40	134.2	8.2
Test 9	40	136.0	8.4
Test 10	40	139.1	8.8

Table 5 shows the mean difficulty and standard deviations of the items in PAT Maths Adaptive upon its release in 2021. Testlets later added to these locations (testlet containers) have similar mean item difficulties and standard deviations.

Table 6 *Mean difficulty and standard deviation of PAT Maths Adaptive testlets*

Testlet location		No. of items	Mean item difficulty (scale score)	Standard deviation (scale score)
Stage 1	Level 1	10	87.3	11.1
	Level 2	10	102.6	10.7
	Level 3	12	109.6	9.3
	Level 4	12	121.7	8.9
	Level 5	12	129.0	8.3
	Level 6	14	134.9	7.0
	Level 7	14	141.5	8.1
	Level 8	14	146.0	8.3
Stage 2	Level 1	10	79.6	8.6
	Level 2	10	96.4	5.7
	Level 3	12	108.4	5.4
	Level 4	12	118.7	5.8
	Level 5	12	126.3	6.5
	Level 6	14	133.0	6.0
	Level 7	14	138.1	5.2
	Level 8	14	142.7	6.0
	Level 9	14	147.9	6.6
Stage 3	Level 1	10	65.6	6.7
	Level 2	10	89.9	6.5
	Level 3	12	102.9	6.2
	Level 4	12	113.1	5.8
	Level 5	12	123.8	5.1
	Level 6	14	130.1	5.3
	Level 7	14	135.0	4.4
	Level 8	14	141.4	5.5
	Level 9	14	146.9	5.8
	Level 10	14	151.2	5.0

Curriculum appropriateness

Learning progression in maths not only requires the ongoing application and refinement of increasingly sophisticated skills but also depends on exposure to, and understanding of, new content and mathematical processes. Because of this closer reliance on the taught curriculum – compared to reading comprehension, for example – it is important that the content assessed by PAT Maths is appropriate for the age and year level of students. The content covered by the items is mapped to the Australian Curriculum, Version 9.0 and care is taken to ensure that students working at the lower levels are not exposed to curriculum content that requires explicit teaching from levels above.

Delivery

Frequency

For the purpose of monitoring student progress, a gap of 9 to 12 months between PAT Maths testing sessions is recommended. Learning progress may not be reflected in a student's PAT Maths scale scores over a shorter period of time. Longitudinal growth should be measured over a minimum of two years of schooling, or three separate testing sessions, in most contexts. This will help account for possible scale score variation, for example where external factors may affect a student's performance on a particular testing occasion. Due to the higher rate of learning progress typically observed in the early years of school, and the greater diagnostic focus of the assessments, it may be appropriate to administer PAT Early Years Maths more frequently.

Choosing the right test

Planning and consistency are important in ensuring PAT Maths is used effectively and that students' results are useful and meaningful. For PAT Maths 4th Edition,, the difficulty of a test and the teacher's knowledge of a student should be taken into consideration when selecting an appropriate level. Curriculum appropriateness and the context of the classroom also need to be taken into account when making this decision. There is often a wide range of ability within the classroom and it is not necessary to provide all students in a class with the same test. Instead the focus should always be each student's ability at the time of the assessment, not where they are expected to be.

The structure of PAT Maths Adaptive removes the need to manually choose and assign test levels to students.

Test administration

Teachers are required to supervise test administration. Practice items are available to support administration of the tests. The recommended test administration time is 40 minutes for PAT Maths Adaptive and PAT Maths 4th Edition and 20–35 minutes for PAT Early Years Maths. This should be sufficient for all students to complete their tests. Consistency in the time allowed to students will assist teachers in comparing the results of students. Teachers may read the questions aloud to students for Test 1 and 2, but not for Tests 3 to 10.

Calculator use

Calculators are not recommended for PAT Early Years Maths tests.

For PAT Maths 4th Edition Tests 1 and 2, and for the last four questions of Tests 3 to 7, calculators should not be used. There are some items in Tests 8 to 10 that require the use of a standard scientific calculator.

At the lower test levels, PAT Maths Adaptive items assess students' abilities to perform simple mathematical operations without the use of a calculator. For a select number of items at levels 4 to 6, an online calculator tool within the test can be used by students, if they choose. The mathematical processes involved in these questions can be completed without the use of a calculator, but students who already feel confident using a calculator may benefit from using one as it will enable them to concentrate on the more complex cognitive aspects of the question.

At upper levels, PAT Maths Adaptive assesses the application of more advanced and complex mathematical processes for which the use of a calculator is appropriate and may in fact be required. The online calculator tool is available for all questions at levels 7 to 10.

Computer Algebra System (CAS) calculators should not be used on any PAT Maths tests.

Item response formats

All items in PAT Maths 4th Edition, and the majority of items in PAT Maths Adaptive, use a selected response format (multiple choice with 3, 4 or 5 options). In PAT Maths Adaptive, constructed response and interactive item types (for example, drag-and-drop, hotspot and matching) are also used for assessing particular skills. Drag-and-drop items are particularly appropriate for assessing the order of values or when a comparison is required. For PAT Early Years the majority of items are drag and drop. Other response types are multiple choice with up to four options, hot spots with 3, 4 or 5 options) and line matches. Examples of different item types are provided in Appendix 3.

PAT Early Years

PAT Early Years Maths is an online assessment, designed for delivery on a tablet and compatible with most desktop computers and laptops. It is supported by audio where appropriate.

Table 7 *Summary of test delivery factors for PAT Early Years Maths*

Test level	Generally suitable for	No. of items	Time allowed
Start Foundation	Foundation	30	20 – 35 minutes
Mid-Foundation	Foundation		
Mid-Year 1	Foundation, Year 1		
End Year 1	Foundation, Year 1		

PAT Maths 4th Edition

PAT Maths 4th Edition, is a suite of linear tests ranging from Test 1 to Test 10 and can be administered according to students' ability, based on their previous scale score as well as the educator's professional judgement.

Table 8 *Summary of test delivery factors for PAT Maths 4th Edition*

Test level	Generally suitable for	No. of items	Time allowed
Test 1	Year 1	30	40 minutes
Test 2	Year 1, Year 2, Year 3	29	
Test 3	Year 2, Year 3, Year 4	35	
Test 4	Year 3, Year 4, Year 5	35	
Test 5	Year 4, Year 5, Year 6	35	
Test 6	Year 5, Year 6, Year 7	40	
Test 7	Year 6, Year 7, Year 8	40	
Test 8	Year 7, Year 8, Year 9	40	
Test 9	Year 8, Year 9, Year 10	40	
Test 10	Year 9, Year 10	40	

PAT Maths Adaptive

A testlet-based adaptive model or multistage testing (MST) is used for PAT Maths Adaptive. A testlet is a small block of items presented to a student. Testlet containers at each stage and level of PAT Maths Adaptive comprise several testlets covering comparable distributions of item content and difficulty. The content and difficulty of testlets in containers are simpler at lower levels and more advanced at higher levels.

Students' entry levels are automatically assigned depending on their estimated ability – according to their most recent PAT score in the learning area and/or their current year level. After completing a testlet at each stage, students are allocated another testlet of items of targeted difficulty based on their cumulative performance to that point. The branching mechanism is designed to provide the maximum amount of information for estimating a student's ability (mean scale score) at each stage after the first testlet.

Testlet containers allow the allocation of different but equivalent content to students according to various system rules. 'Parallel' testlet rules mean that students are presented alternative content in consecutive PAT Maths Adaptive test sittings, reducing their potential exposure to the same content. Testlet containers also support the delivery of 'accessible' items to students requiring test content that is compatible with screen readers or other assistive technologies. If a student is assigned an accessible PAT Maths Adaptive test, they will only be presented testlets containing items that meet the Web Content Accessibility Guidelines (WCAG) 2.1 AA Standard. Parallel accessible testlets are not currently available.

Figure 1 displays the PAT Adaptive MST design for a panel at an entry level. There are eight entry levels for PAT Maths Adaptive. At each entry level between levels 2 and 6, a panel of MST consists of eleven testlet containers and thirteen pathways. Figure 2 displays the overall PAT Maths Adaptive test design consisting of eight entry levels for increasing levels of item difficulty/student ability.

Figure 1 *Panel design for PAT Maths Adaptive at an entry level, including parallel and accessible testlets*

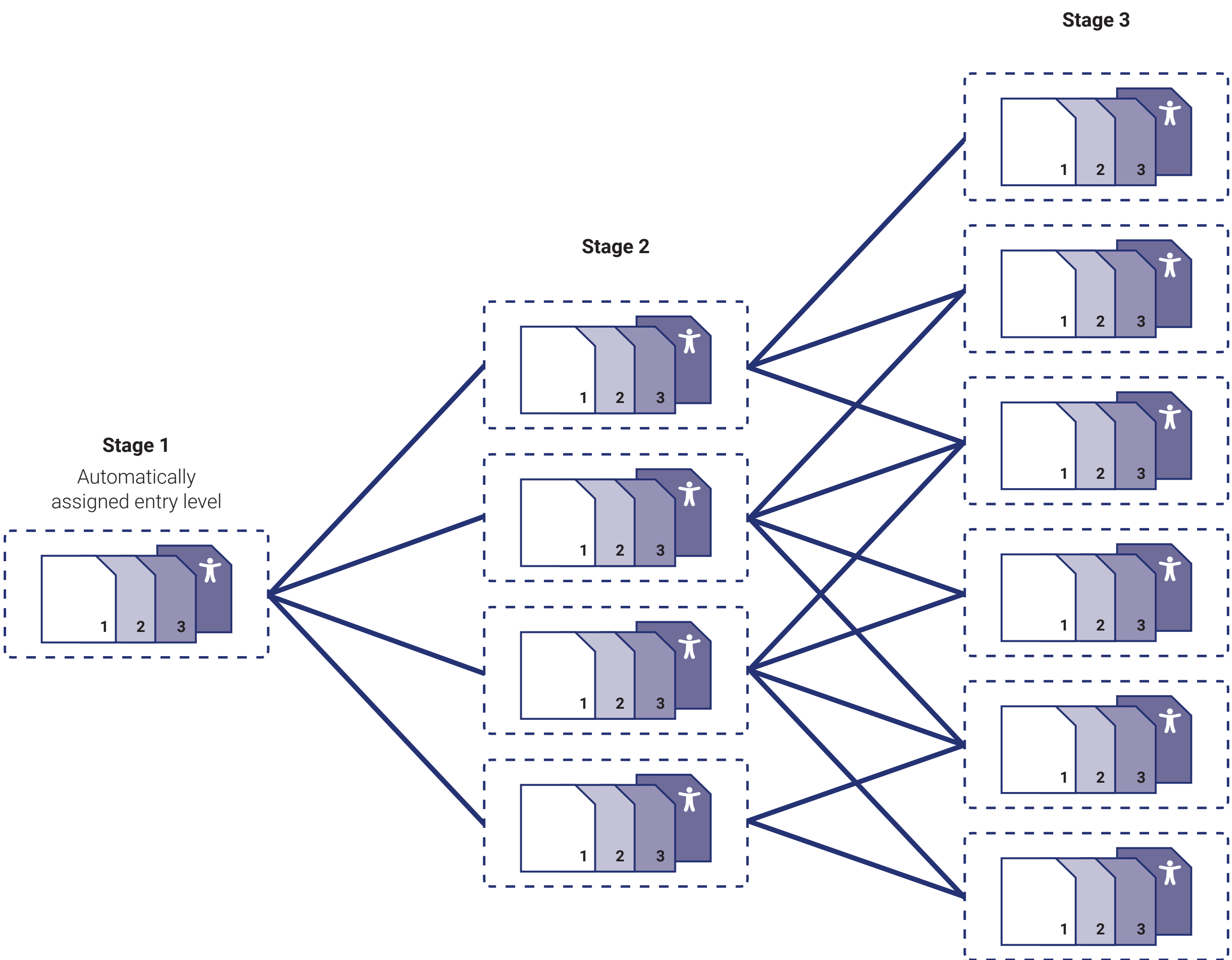
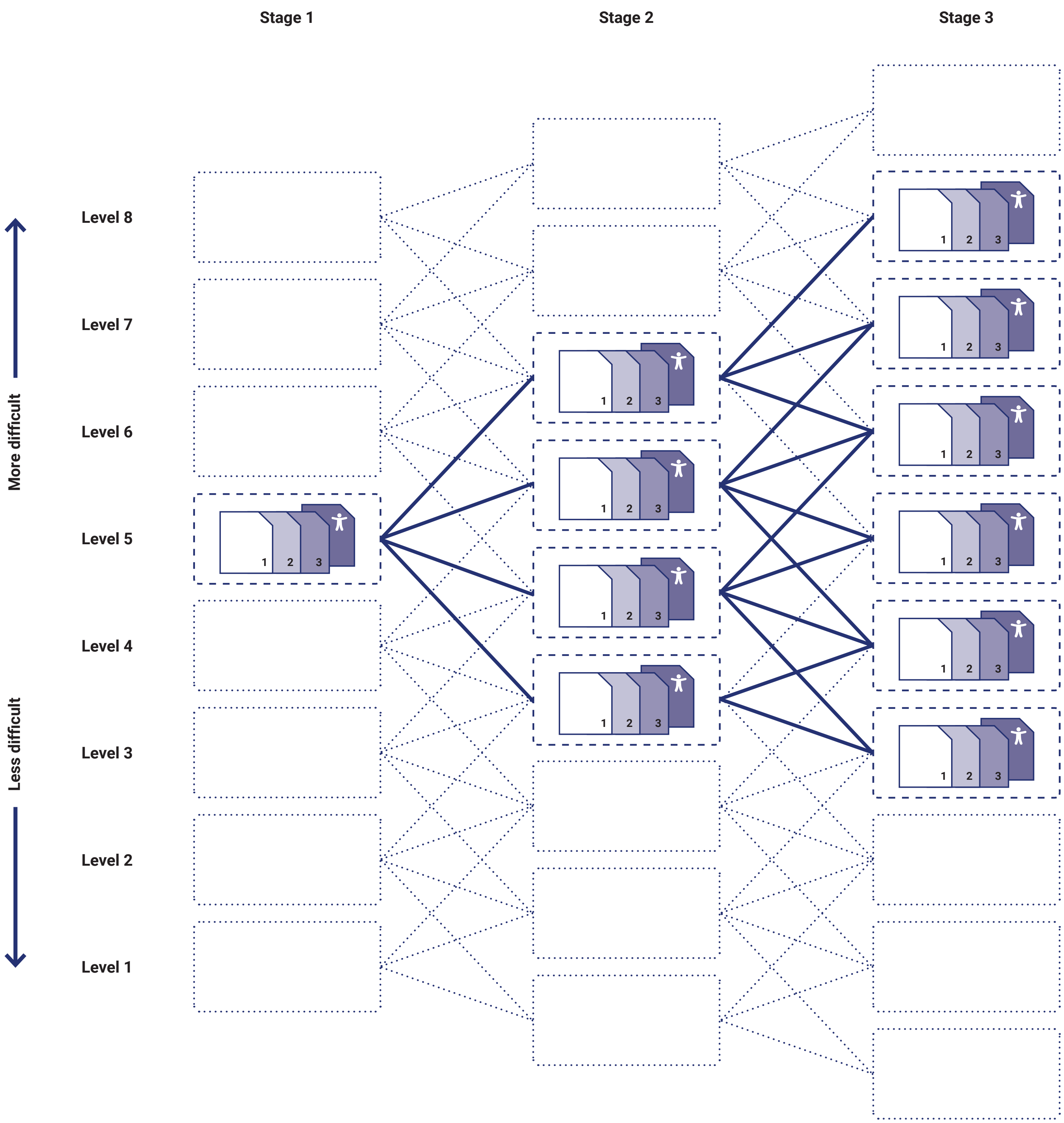


Figure 2 *PAT Maths Adaptive test design with eight entry levels*



Reporting

The information provided by the PAT Maths reports is intended to assist teachers in understanding their students' abilities in mathematics, diagnosing gaps, strengths and weaknesses in students' learning, and measuring learning progress over time.

PAT scale score

A PAT scale score is a numerical value given to a student whose achievement has been measured by completing a PAT assessment. A student's scale score lies at a point somewhere on the specific PAT scale, and it indicates that student's level of achievement in that particular learning area – the higher the scale score, the more able the student.

Regardless of the test level or items administered to students, they will be placed on the same scale for the learning area. This makes it possible to directly compare students' achievement and to observe students' progress within a learning area by comparing their scale scores from multiple testing periods over time.

Item difficulty is a measure of the extent of skills and knowledge required to be successful on the item. This makes it possible to allocate each PAT Maths test item a score on the same scale used to measure student achievement. An item with a high scale score is more difficult for students to answer correctly than a question with a low scale score. It can generally be expected that a student is able to successfully respond to more items whose difficulty is located below their achieved scale score than above. By referencing the difficulty of an item, or a group of items, and the proportion of correct responses by a student or within a group, it may be possible to identify particular items, or types of items, that have challenged students.

A score on the PAT Maths scale has no meaning on the PAT Reading scale. In fact, the units of the scale will have different meanings for each scale. This is because these units are calculated based on the range of student levels of achievement, which vary widely between learning areas.

Achievement bands

While a scale score indicates a student's achievement level, and can be used to quantitatively track a student's growth, it is only in understanding what the number represents that teachers can successfully inform their practice to support student learning. For this reason, the PAT scale has been divided into achievement bands that include written descriptions of what students are able to do at that band (band description). A student scoring in a particular band can be expected to have some proficiency in that band and be progressively more proficient with the Maths knowledge, skills and understanding outlined in lower bands.

Students in the same achievement band are operating at approximately the same achievement level within a learning area regardless of their school year level. Viewing student achievement in terms of achievement bands may assist teachers to group students of similar abilities. By referencing the PAT achievement band descriptions, teachers can understand the types of skills typical of students according to their PAT band.

A PAT Maths scale score of 105 could be considered to be at the upper end of achievement band 95–104 or at the lower end of achievement band 105–114. In cases like these, it is important to reference the descriptions of both achievement bands to understand the student's abilities.

The achievement band descriptions for PAT Maths are elaborated by strand and can be found within ACER's online School Support Centre.

The following extract shows the band descriptions for the PAT Maths sub-strand Statistics.

PAT Maths achievement band descriptions ***Statistics sub-strand***	
155 and above	Students in this band typically are able to analyse statistical information of various types to justify an opinion or evaluate a statement.
145–154	Students in this band typically are able to interpret and synthesise data represented in different ways, such as in tables (two-way tables and grouped data), grouped column graphs, histograms, line graphs and scatter plots. They can also calculate, interpret and use measures of central tendency and dispersion from a variety of data sources including grouped data.
135–144	Students in this band typically can retrieve data from a variety of statistical representations, including line graphs, box plots, stem-and-leaf plots, segmented (stacked) column graphs, frequency tables of grouped data and two-way tables. They can calculate and use the mean, median and mode for ungrouped data in a variety of contexts. They can also compare two sets of data to solve problems or draw conclusions.
125–134	Students in this band typically are able to interpret two-way tables, column graphs, line graphs and pie charts involving percentage values. They can evaluate conclusions provided to determine whether they are supported by given data. They can identify relationships within displays of bivariate data, and can calculate and use summary statistics (for example, measures of central tendency and dispersion) for ungrouped data. They can recognise the factors that relate to selection of a representative sample for a data collection exercise.
115–124	Students in this band typically are able to calculate the mean (average) and range of a data set and solve problems involving the mean. They can retrieve and interpret data from tables, column graphs (including two-category column graphs and those involving grouped data) and pictographs (including those that use a symbol for multiple and fractional units and half symbols), and can identify different representations of the same data set. These students can also evaluate a small set of given survey sampling methods to identify the method that best meets the survey purpose.
105–114	Students in this band typically are able to design simple survey questions to gather data. They can organise given or collected data into an appropriate representation such as a pictograph, column graph or table. Students can also retrieve and interpret data displayed as: a pictograph where the symbol represents more than one unit; a two-category column graph or a column graph with a vertical axis that represents a measured quantity rather than a frequency; and a two-way table or tally chart.
95–104	Students in this band typically are able to represent category counts as a tally chart, pictograph or column graph. They can also interpret column graphs and pictographs, where the symbol represents more than one unit, to solve problems that involve consideration of two or more data categories.
85–94	Students in this band typically are able to represent category counts (in single digits) as a tally chart or pictograph. They can also interpret simple graphs, tally charts and pictographs to solve problems (for example, to calculate a total represented by several rows on a tally chart).
75–84	Students in this band typically are able to retrieve information from a simple graph or tally chart to identify the number in a specified category (single digits). They can also compare data values represented in a simple graph to draw an inference such as the least or the greatest.
65–74	Students in this band typically are able to classify and sort familiar objects into groups according to simple attributes (for example, colour, number of legs and type of toy). Students begin to develop subsequent skills in this sub-strand (beyond grouping and sorting) at a higher band level.
64 and below	Students in this band typically are able to classify and sort familiar objects into groups according to simple attributes (for example, colour, number of legs and type of toy).

While the achievement band descriptions are intended to be considered in their entirety and not as discrete components, these extracts help to demonstrate the progression of particular skills. In 'typical' development of mathematical ability, students progress from being able to recognise small numbers and perform simple operations, to being able to apply mathematical ideas to real-world contexts, and interpret the mathematics that is embedded in various unfamiliar settings. Knowing at which stage a student's ability is located can help target learning for students performing at these different levels, to ensure their progression from one level to the next.

Norms

PAT Maths norm data that represents the achievement of students across Australia is available as a reference against which student achievement can be compared. The comparison between a student's scale score achievement and the Australian norms can be expressed as a percentile or stanine ranking.

Percentiles

The percentile rank of a score is the percentage of students who achieve less than that score. Percentiles are useful when measuring the performance of a student against the norm for that year level. For example, a student with a percentile rank of 75th compared to the Year 3 norm has a scale score that is higher than 75% of Australian Year 3 students.

Stanines

Stanines are ranking scores from 1 to 9 derived from the Australian norms. Stanine scores group together percentile ranks to simplify the comparison of student achievement with the norms.

Stanine	Corresponding percentile ranks
9	96th and above
8	90th–95th
7	77th–89th
6	60th–76th
5	40th–59th
4	23rd–39th
3	11th–22nd
2	4th–10th
1	3rd and below

Stanines are not reported for PAT Maths Adaptive as scale scores are a much more appropriate measure of growth. As scale scores are a measurement of what students can and cannot do, rather than a comparison between students, they are a more effective way to target teaching and learning resources.

Appendixes

Appendix 1

Literature review: locating PAT Maths in the broader research context

The dominance of literacy and numeracy within the educational landscape in recent decades, has led to a large body of research on and around mathematics education (Inglis and Foster, 2018; Kilpatrick, 2014). In recent years, the research has gathered around several questions:

- What is the difference between mathematics and numeracy, and how does this relate to teaching?
- What are the cognitive skills involved in mathematics, and what does this mean for classroom teaching and effective learning?
- What role does technology play in the teaching and learning of mathematics?
- What are the most effective assessment practices in mathematics and how can they support teaching and learning?

The development and introduction of the Australian Curriculum has allowed for some of the ideas in mathematics education research to coalesce. Further, the implementation of an assessment program that is standardised nationally, the National Assessment Program in Literacy and Numeracy (NAPLAN), has ensured agreement on topics such as the definition of numeracy and prompted further research around the role of assessment in the classroom.

Mathematics and numeracy

'Mathematics' is a domain of learning within the Australian Curriculum, but 'Numeracy' is also included in the curriculum as a general capability. The Australian Curriculum describes numeracy as students being able 'to use mathematics confidently across other learning areas at school and in their lives more broadly' (ACARA, 2010). The Australian Curriculum, Version 9.0 states: 'Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully (ACARA, 2017). Sullivan (2011) points out that the inclusion of Numeracy means that other learning areas can be brought into the mathematics classroom and explored through the lens of mathematics. This fits with other accepted definitions of numeracy, such as that of the Australian Association of Mathematics Teachers (AAMT), which describes numeracy as involving '… the disposition to use, in context, a combination of underpinning mathematical concepts and skills from across the discipline; mathematical thinking and strategies; general thinking skills; [and] grounded appreciation of context' (AAMT, 1997). They point out that although mathematics and numeracy are not identical, all numeracy is underpinned by mathematics and therefore school mathematics at the compulsory levels is essential to being numerate. The Victorian Curriculum and Assessment Authority expanded upon the AAMT definition: Numeracy is the knowledge, skills, behaviours and dispositions that students need in order to use mathematics in a wide range of situations (Department of Education and Training (Victoria), 2017).

School mathematics at the compulsory levels should focus primarily on 'practical and useable mathematics that can enrich not only students' employment prospects but also their ability to participate fully in modern life and democratic processes' (Sullivan, 2011). This need to be numerate will only become more essential in the future (Rubenstein, 2009).

Cognitive skills

Mathematics can be naturally classified according to content. The topics of Number, Algebra, Measurement, Geometry, Statistics and Probability are common and conventional. Although the names may be different (Data, Chance, Space and Shape are all commonly used alternatives) and the focus may vary, all curricula are structured generally around these and they are used to design and report on assessment. The 'what' of mathematics changes little between curricula over time and between countries, but the 'how' of mathematics curricula has become much more fluid.

Mathematics demands different cognitive skills according to how it is being applied. Prior to the development of a national curriculum, most Australian jurisdictions had courses of study that used the term 'working mathematically' to describe the way that mathematics was performed. In developing the Australian Curriculum, ACARA (2010a) argued that 'the notion of 'working mathematically' creates the impression to teachers that the actions are separate from the content descriptions, whereas the intention is that the full range of mathematical actions apply to each aspect of the content.' As an alternative, they proposed that the content be delivered around four 'proficiency strands', which can be thought of as the verbs (the nouns being the three content strands). These were adapted from the proficiency strands proposed in the 2001 research review by the (American) National Research Council (Kilpatrick et al, 2001). In that review, the authors determined that procedural fluency in mathematics was the basis for most teaching programs. This resulted in producing students who were most proficient in procedural fluency but less able to solve problems, apply deductive reasoning and make links between the different areas of mathematics. They proposed that the curriculum be designed around five interwoven and interdependent proficiencies, able to be demonstrated in some way at every level of schooling. Four of these (Understanding, Fluency, Problem solving, and Reasoning) now constitute the proficiency strands of the Australian Curriculum: Mathematics F–10.

Technology

The Report of the Review to Achieve Educational Excellence in Australian Schools (Gonski et al, 2018), states that it should be a priority that 'every child should have the skills and knowledge to be active participants in a rapidly changing world of technology'. Initiatives such as STEM (Science, Technology, Engineering, and Mathematics) have been developed in support of this idea and interest in and around STEM education is increasing in importance around the world (Li et al, 2020).

The most common technological aid in mathematics classrooms is the calculator. Since their introduction, the use of calculators has been a source of disagreement between educators and policy makers. Early studies in the UK (Bell et al, 1978; Girling, 1977) discovered certain skills that were important if calculators were to be used effectively in the lower years. These included single-digit arithmetic, an understanding of place value and an awareness of the reasonableness of answers. Later studies confirmed that in order for a calculator to be effective as a learning tool, an understanding of number was required (McIntosh, 1997). There is evidence that having access to calculators can improve young children's mathematical ability and enhance their understanding of the skills involved (Ruthven, 1998; NCTM, 2015; Ellington, 2003).

The use of calculators in assessments is considered less beneficial. Caution should be exercised when assessing certain aspects of mathematics with and without calculators (Bridgeman et al, 1995). Complex arithmetic processes become much less so when a calculator is used, and so the construct of any assessment should be considered carefully depending on whether or not calculators can be used. In Australia, recent large-scale assessments have overcome the problem by having calculator-free and calculator-allowed components. Although it is acknowledged that calculators are a commonplace and supportive tool for learning, NAPLAN (ACARA, 2018) does not permit the use of calculators for the Numeracy assessment at Years 3 and 5. At Years 7 and 9 there is a set of items assessing mental calculation skills at the start of the assessment, for which calculator use is not allowed.

Early years mathematics

Being numerate is the capacity, confidence, and disposition to use mathematics in daily life. Young children are naturally curious about their environment and through experiences in early childhood instinctively problem solve different situations.

To build numeracy, children explore powerful mathematical ideas in their world including spatial sense, geometric and algebraic reasoning, structure and pattern, number sense, data and probability reasoning and measuring, along with drawing connections and argumentation (Clements & Sarama, 2007; Ginsberg et al., 2006; Starkey et al., 2004; Samara & Clements, 2008). This literature review identified the importance of teaching particular concepts in the early primary school years to build a foundation for future mathematics learning and thereby guide inclusion of material in the PAT Early Years Framework.

Early mathematical knowledge and skills develop rapidly during the pre-school years. Therefore, children enter formal schooling with a variety of early numeracy skills (Klibanoff et al., 2006, Reid, 2016). Research by Jordan et al. (2009) identified a significant link between early number competence and mathematics achievement in later grades at primary school. Klibanoff et al. (2006) affirm this by suggesting that a child's exposure to early numeracy concepts can affect how they acquire more complex mathematical concepts in their later years of schooling. Data from the work of Wijns et al. (2020) suggests there is a tendency for children to spontaneously attend to mathematical elements in their environment.

The USA National Research Council (NRC) report (2009) provided a comprehensive review of research, laying out the critical areas that should be the focus of young children's early mathematics education to improve the quality of mathematics experiences for young children These mathematical concepts - pattern, number, measurement and geometry and data and probability are also highlighted in the findings of other researchers, and practitioners (ACARA 2023; Clarke et al., 2002; Klein et al, 1999; Markovits, 2019).

Pattern

Patterning includes the ability to notice and use predictable sequences and spatial skills (Zippert, 2019). It has been found that these skills are both predictors of children's future math knowledge (Lüken & Kampmann, 2018; Rittle-Johnson et al., 2019; Mulligan & Mitchelmore, 2018). There are different types of patterns, including repeating patterns and growing patterns. The focus, both in research and in practice with preschoolers, has been on repeating patterns, which are the most easy and accessible for this age group (Wijns et al., 2020).

Number

A fundamental skill in early childhood is learning the names of numbers, matching these names to their symbols and then to groups of objects (NRC, 2009). This is referred to as number sense. Yilmaz, (2017) explores three key areas of number sense and identifies these as number knowledge, counting and arithmetic operations. Children's competency with number sense in the early years is predictive of future mathematics success particularly in the upper primary years (Fuchs et al., 2007; Jordan et al., 2009, NRC, 2009. Torbeyns et al., 2015). Before counting is consolidated children compare sets of objects to identify which set has more (NRC, 2009). Oral counting then develops in stages from individual words through forwards and backwards sequences to skip counting. The counting concepts, identified by Gelman and Gallistel (1978), are one-to-one correspondence, stable-order, cardinality, abstraction and order-irrelevance. These principles have continued to be reinforced by later research.

Children move through a progression of strategies to perform calculations, from using physical objects to using mental strategies. The Early Numeracy Research Project (ENRP) (1999) identified a series of growth points which named and described these strategies, including count all, count on, count back, count down to, and count up from, for addition and subtraction (Clarke et al, 2002).

Measurement and space

The early development of spatial and geometry skills can predict school success in mathematics, science and technology (Mix, et al., 2016). Therefore, it is important for children to be exposed to opportunities where they can develop these skills before they attend formal schooling.

The developmental sequence of strategies and devices used by children in measurement tasks was explored by Boulton et al. (1996). They suggested that children should be encouraged to measure directly and indirectly with both standard measures and arbitrary units from the first year of school. An emphasis on measurement allows children to use numbers in relevant and meaningful ways (Cheeseman et al., 2018).

Recent work by Outhwaite et al. (2019) focused on geometry - lines, patterns and shape. They identified several content areas which should be taught to children in the early years, including comparison, shape selection, position, everyday language related to time, sequencing events and two-dimensional shapes. Cohrssen et al. (2017) confirmed that shape selection (i.e. the investigation of two- and three-dimensional shapes) supported children's spatial thinking skills.

Data and probability

It is evident from this literature review that is a lack of research on data and probability concepts being taught in the early years. The Australian Curriculum, Version 9.0 includes teaching statistics from year 1. It then introduces probability in year 3. There is, however, an interconnectedness between the mathematical skills outlined in this review. This is evidenced in how the use of number concepts is integral to accurate data collection and recording. Sorting and classifying of objects are introduced in early childhood classrooms albeit this process is informal but can support recording of data with pictures or marks for each object. With scaffolding and appropriate support, young children can also engage in mathematical tasks where they can respond to simple surveys and make observations on the results. Leavy and Hugen (2018) considered representations and explanations of data to be important aspect of early childhood mathematics due to how these skills build on from the sorting and quantifying of groups.

Effective assessment

The connection between curriculum, practice and assessment must be strong for any one element to be effective (OECD, 2013) and so it is important that assessment is based on what is taught and that the results of any assessment inform the teaching that follows. Assessment should be more than just the instrument; it includes 'the process of drawing inferences from the data collected and acting upon those judgements in effective ways' (Callingham, 2010, p 39). Clarke (1996) suggests that teachers should ask themselves, 'how will this assessment promote and inform subsequent action by me, by other teachers, by my students and by parents or other members of the community?'

Masters (2013, p 6) states that 'the fundamental purpose of assessment is to establish where learners are in their learning at the time of assessment'. In order to be most effective at doing this, it is important that any assessment allows teachers to provide timely and accurate feedback (Hattie, 2009). Assessment should be appropriate, fair and inclusive and should inform learning and action (AAMT, 2008).

Gonski (2018) reported that there was 'compelling evidence, in Australian schools and internationally, that tailored teaching based on ongoing formative assessment and feedback is the key to enabling students to progress to higher levels of achievement'. This supports the findings of Black and Wiliam (1998, 2003) who wrote extensively of the positive effects of good quality formative assessment on learning.

In terms of formative, diagnostic assessment, a wide variety of tools are used in Australian classrooms, including tests, open-ended, rich tasks and learning progressions (Forster, 2009). Although summative assessment is also used in Australian classrooms (for instance NAPLAN and international tests such as PISA and TIMSS), these too are often used to inform teaching and learning. As Black and Wiliam (2009) make clear, 'any assessment is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction'. (p 9) These assessments, although usually thought of and used at the school or system level, can often be unpacked at the item level as well, making them valuable formative tools (Siemon, 2019; Callingham 2010).

Another way that assessment can be more effective is the use of computer adaptive tests. Not only are these more precise at measuring student achievement, but they have the added benefit that students are more highly motivated and engaged while completing them (Martin & Lazendic, 2018). When being assessed, there is a greater likelihood that the challenge of the task is appropriate for their ability level; within the zone of proximal development (Vygotsky, 1978). Computer adaptive tests are also able to present items to candidates that are slightly below and above their level of ability. There is evidence that this improves test accuracy and efficiency for some students (Wei & Lin, 2015).

In the future, it is probable that assessment of and for the learning of mathematics will develop along the paths described above. The recommendations made by Gonski (2018, Recommendation 4) include that reporting on assessment have an emphasis on achievement and growth, and that the growth should be measured against learning progressions. Masters (2013) also expresses the idea that learning should be assessed by measuring growth over time and against empirically derived learning progressions.

Works cited

Australian Association of Mathematics Teachers [AAMT]. (1997). Numeracy=everyone's business. *The Report of the Numeracy Education Strategy Development Conference*, May 1997. Adelaide: AAMT.

Australian Association of Mathematics Teachers [AAMT]. (2008). *Position paper on the practice of assessing mathematics*. Adelaide: Author https://aamt.edu.au/wp-content/uploads/2020/07/Assessment_position_paper_2017.pdf

Australian Curriculum Assessment and Reporting Authority [ACARA]. (2010a). *The shape of the Australian Curriculum: Mathematics*. Accessed December 1, 2010. http://www.acara.edu.au/phase_1_-_ the_australian_curriculum.html

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2014). Foundation to year 10 curriculum: General Capabilities (Numeracy). https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy/

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2014). *Foundation to year 10 curriculum: Mathematics*. https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/

Australian Curriculum Assessment and Reporting Authority [ACARA]. (2018). *The Australian National Assessment Program Literacy and Numeracy (NAPLAN) assessment framework: NAPLAN Online 2017-2018*. https://www.nap.edu.au/docs/default-source/default-document-library/naplan-assessment-framework.pdf?sfvrsn=2

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2023). Foundation to year 10 curriculum: General Capabilities (Numeracy). https://v9.australian curriculum.edu.au/teacher-resources/understand-this-general-capability/numeracy

Bell, A., Burkhardt, H., McIntosh A.J. & Moore, G., (1978). *A Calculator Experiment in a Primary School. Shell Centre for Mathematical Education*. Nottingham, UK.

Black Paul, & Wiliam Dylan. (1998). Inside the Black Box: Raising Standards through Classroom Assessment. *The Phi Delta Kappan, 80(2)*, 139–148.

Black Paul, & Wiliam Dylan. (2003). In Praise of Educational Research: Formative Assessment'. *British Educational Research Journal, 29(5)*, 623–637.

Black, P., Wiliam, D. Developing the theory of formative assessment. Educ Asse Eval Acc 21, 5–31 (2009). https://doi.org/10.1007/s11092-008-9068-5

Boulton-Lewis, G. M., Wilss, L. A., & Mutch, S. L. (1996). An analysis of young children's strategies and use of devices for length measurement. *The Journal of Mathematical Behavior, 15*(3), 329-347.

Bridgeman, B., Harvey, A., & Braswell, J. (1995). Effects of Calculator Use on Scores on a Test of Mathematical Reasoning. *Journal of Educational Measurement, 32(4)*, 323–340. Retrieved April 21, 2021, from http://www.jstor.org/stable/1435216

Callingham, R. (2010). Mathematics assessment in primary classrooms. In ACER (Ed.), *Proceedings of 2010 ACER Research Conference*, 39–42, Melbourne: ACER. https://research.acer.edu.au/cgi/viewcontent.cgi?article=1069&context=research_conference

Cheeseman, J., Benz, C., & Pullen, Y. (2018). Number sense: The impact of a measurement-focused program on young children's number learning. *Contemporary research and perspectives on early childhood mathematics education*, 101-127.

Clarke, D. J. (1996). Assessment. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education*, 327–370, Dordrecht, The Netherlands: Kluwer.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., McDonough, A., ... & Rowley, G. (2002). Early numeracy research project (ENRP): Final report.

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks project. *Journal for Research in Mathematics Education, 38*(2), 136-163.

Cohrssen, C., de Quadros-Wander, B., Page, J., & Klarin, S. (2017). Between the big trees: A project-based approach to investigating shape and spatial thinking in a kindergarten program. *Australasian Journal of Early Childhood, 42*(1), 94-104.

Department of Education and Training (Victoria). (2017). Literacy and Numeracy Strategy Phase 2. https://www.education.vic.gov.au/Documents/school/teachers/teachingresources/literacynumeracy/Literacy_and_Numeracy_Strategy_Phase_2.pdf

Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education*, 433–463.

Forster, M., & Australia. Dept of Education, E. and W. R. (DEEWR) (Eds). (2009). Literacy and numeracy diagnostic tools: an evaluation. Commonwealth Department of Education, Employment and Workplace Relations (DEEWR). https://research.acer.edu.au/cgi/viewcontent.cgi?article=1017&context=monitoring_learning

Fuchs, L. S., Fuchs, D., Compton, D. L., Bryant, J. D., Hamlett, C. L., & Seethaler, P. M. (2007). Mathematics screening and progress monitoring at first grade: Implications for responsiveness to intervention. *Exceptional children, 73*(3), 311-330.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Harvard University.

Ginsburg, H.P., Goldberg Kaplan, R., Cannon, J., Cordero, M.L., Eisenband, J.G., Galanter, M., and Morgenlander, M. (2006). Helping early childhood educators to teach mathematics. In M. Zaslow and I. Martinez-Beck (Eds.), Critical Issues in Early Childhood Professional Development (pp. 171-202). Baltimore: Paul H. Brook

Girling, M. (1977). Towards a definition of basic numeracy. *Mathematics Teaching*, 81, 4–5, 13–14.

Gonski, D., Arcus, T., Boston, K., Gould, V., Johnson, W., O'Brien, L., Perry, L.-A., Roberts, M., & Australia. Dept of Education (Eds). (2018). *Through growth to achievement: report of the review to achieve educational excellence in Australian schools.*

Hattie, J.A.C. (2009). *Visible learning: a synthesis of meta-analyses relating to achievement*. Abingdon: Routledge.

Inglis, Matthew & Foster, Colin. (2017). Five Decades of Mathematics Education Research. *Journal for Research in Mathematics Education*. 49. 10.5951/jresematheduc.49.4.0462.

Johnston, J. R. (1984). Acquisition of locative meanings: Behind and in front of. *Journal of Child Language, 11*(2), 407-422.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*(3), 850.

Kilpatrick J. (2014) History of Research in Mathematics Education. In: Lerman S. (Eds) *Encyclopedia of Mathematics Education*. Springer, Dordrecht. https://doi.org/10.1007/978-94-007-4978-8_71

Klein, A., Starkey, P., & Wakeley, A. (1999, April). Enhancing prekindergarten children's readiness for school mathematics. Paper presented at the Annual Meeting of the American Educational Research Association

Li, Y., Wang, K., Xiao, Y. et al. (2020). Research and trends in STEM education: a systematic review of journal publications. *IJ STEM Ed 7, 11* (2020). https://doi.org/10.1186/s40594-020-00207-6

Lüken, M. M., & Kampmann, R. (2018). The influence of fostering children's patterning abilities on their arithmetic skills in grade 1. Contemporary research and perspectives on early childhood mathematics education, 55-66.

Martin, A. J., & Lazendic, G. (2018). Computer-Adaptive Testing: Implications for Students' Achievement, Motivation, Engagement, and Subjective Test Experience. *Journal of Educational Psychology, 110(1)*, 27–45. ERIC.

Markovits, Z. (2019). Children's Ways of Thinking When Coping with Everyday Mathematical Situations. Educatia Plus, 24, 185–190.

Masters, G., (2013) Reforming education assessment: imperatives, principles and challenges 2013, *Australian Education Review*, No. 57.

Mix, K. S., Levine, S. C., Cheng, Y. L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General, 145*(9), 1206.

McIntosh, A., (1990). Becoming Numerate: Developing Number Sense in Being Numerate: What Counts? Ed. Willis, S. The Australian Council for Educational Research: Australia

Mulligan, J., & Mitchelmore, M. (2018). Promoting early mathematical structural development through an integrated assessment and pedagogical program. *Contemporary research and perspectives on early childhood mathematics education*, 17-33.

National Research Council. (2009). Mathematics Learning in Early Childhood: Paths Towards Equity and Excellence. Washington, DC: The National Academies Press.

National Council of Teachers of Mathematics (2015), Calculator Use. https://www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Calculator-Use/

Kilpatrick, J., Swafford, J., Findell, B., & National Research Council (U.S.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

OECD, & OCDE (2013). Synergies for Better Learning: *An International Perspective on Evaluation and Assessment*. Éditions OCDE / OECD Publishing. https://doi.org/10.1787/9789264190658-en

Outhwaite, L. A., Faulder, M., Gulliford, A., & Pitchford, N. J. (2019). Raising early achievement in math with interactive apps: A randomized control trial. *Journal of Educational Psychology, 111*(2), 284.

Reid, K. (2016). *Counting on it: Early numeracy development and the preschool child*. Australian Council for Educational Research (ACER). https://research.acer.edu.au/learning_processes/19

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly, 46*, 166-178.

Roche, A., Ferguson, S., Cheeseman, J., & Downton, A. (2020). Making equal groups: The case of 12 little ducks. *Australian Primary Mathematics Classroom, 25*(4), 31–34. https://doi.org/10.3316/informit.654464501099469

Rubinstein, H (2009). A national strategy for mathematical sciences in Australia, National Committee for the Mathematical Sciences. https://www.science.org.au/files/userfiles/support/reports-and-plans/2009/national-strategy-for-math-sciences-in-australia.pdf

Ruthven, K., (1998). The use of mental, written and calculator strategies of numerical computation by upper-primary pupils within a 'calculator-aware' number curriculum. *British Educational Research Journal 24 (1)*, 21–42.

Sarama, J., & Clements, D. H. (2008). Mathematics in early childhood. *Contemporary perspectives on mathematics in early childhood education*, 67-94.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*(1), 99–120.

Siemon, D. (2019), Applying and building on what we know: Issues at the intersection of research and practice. In G. Hine, S. Blackley, & A. Cooke (Eds). *Mathematics Education Research: Impacting Practice (Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)*, 49–67. Perth: MERGA.

Sullivan, P. (2011). Teaching Mathematics: Using research-informed strategies. ACEReSearch.

Torbeyns, J., Gilmore, C., & Verschaffel, L. (2015). The acquisition of preschool mathematical abilities: Theoretical, methodological and educational considerations. *Mathematical Thinking and Learning, 17*(2-3), 99-115.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, Mass.: Harvard University Press.

Wei, H., & Lin, J. (2015). Using Out-of-Level Items in Computerized Adaptive Testing. *International Journal of Testing*, 15(1), 50–70. ERIC.

Wiliam, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree Press.

Wijns, N., De Smedt, B., Verschaffel, L., & Torbeyns, J. (2020). Are preschoolers who spontaneously create patterns better in mathematics? *British Journal of Educational Psychology, 90*(3), 753–769.

Yılmaz, Z. (2017). Young children's number sense development: Age related complexity across cases of three children. *International Electronic Journal of Elementary Education, 9*(4), 891-902.

Zippert, E. L., Clayback, K., & Rittle-Johnson, B. (2019). Not just IQ: Patterning predicts preschoolers' math knowledge beyond fluid reasoning. *Journal of Cognition and Development, 20*(5), 752–771.

Appendix 2

Trial design and assessment validity

A test is said to be valid if it measures what it was intended to measure. The PAT Maths tests are planned and constructed to assess the areas of mathematical ability that are common across curricula and accepted as important by educators. In constructing the tests, care is taken to include a range of all appropriate characteristics to ensure that the breadth of students' abilities can be captured. All of the items are subjected to intensive scrutiny, review and revision by panels of experts.

All items are developed by a group of experienced test developers specialising in Mathematics, who review and panel the items until they are ready for trial. A rigorous process of quality checking, proofreading and formatting then takes place. The psychometric team provides a trial design based on the items (number, distribution of strand and proficiencies). To ensure the most valid and reliable psychometric data is made available, PAT Maths items are currently trialled in standalone trial test forms, with sets of items placed into different locations in multiple versions of a single form. Schools using any of ACER's online PAT assessments may opt for their students to participate in the PAT Maths test trials. Data analysis is performed after trial by the psychometric team, with the Test Development Manager making final decisions about item performance and deletions. Each trial is designed to strengthen the PAT Maths construct, ensuring a spread of items along the PAT scale that meets the demands of strand and proficiency distribution.

In the course of selecting testlets for PAT Maths Adaptive, the best-performing material was retained from the linear forms of the tests. New items were trialled for PAT Maths Adaptive and the best-performing new material from the trial was selected.

The calibration procedures identified items that also appeared to be measuring skills other than those measured by the majority of items. Items 'misfitting' in this way were not retained. The items retained for PAT Maths Adaptive were shown to fit the Rasch measurement model satisfactorily. Items that were not able to discriminate between high and low performing students were not selected for PAT Maths Adaptive. All selected items could be regarded as measuring a student's location on a single underlying continuum of mathematical ability.

PAT Maths items have been trialled across many years, with only successfully performing items becoming part of the PAT Maths item pool for selection in final linear forms and adaptive testlets.

Appendix 3

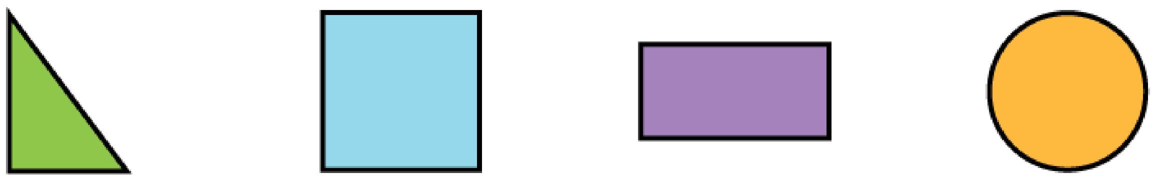
PAT Maths item response format examples

These examples illustrate three commonly used item response formats used in PAT Maths assessments: from top to bottom, simple multiple-choice, hotspot, cloze, and drag-and-drop.

Which number is one less than 8?

- 6
- 7
- 8
- 9
- 10

Which one of these shapes is a circle?



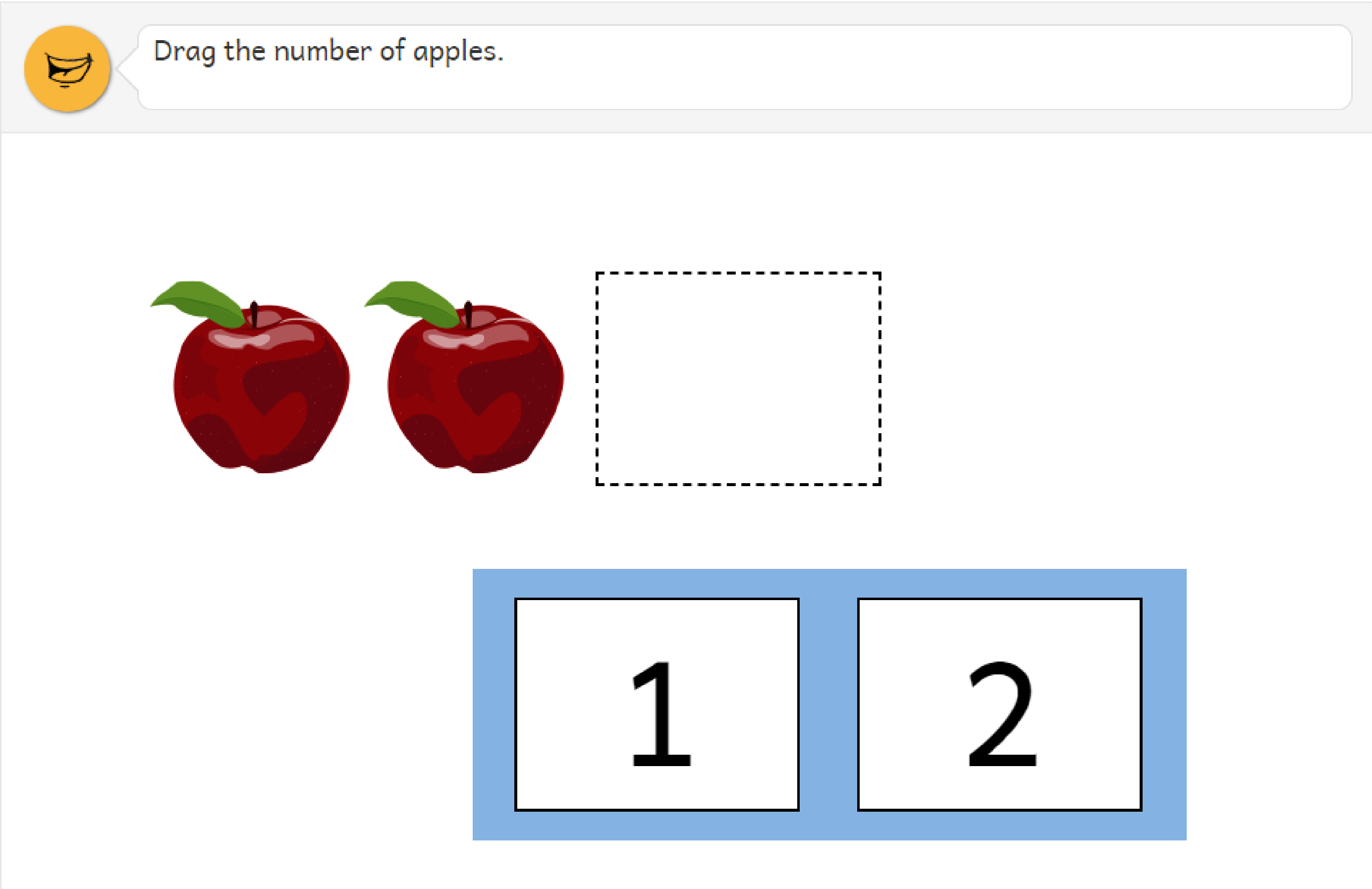
A farmer has five cows.

Each day, one cow eats 40 kilograms of feed and gives 20 litres of milk.

In the morning, the farmer gives each cow **half** of its daily feed.

How much feed does the farmer give each cow in the morning?

______ kilograms


Drag the number of apples.
1
2